Pasha is also against "tails"?

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Solution — it is necessary to reduce each element to the median of the array. That is, if the median is x, then the answer is $\sum_{i=1}^{n} |a_i - x|$.

Proof:

Let's assume that the array a is sorted. Let's fix some k and prove that if we reduce the elements of the array to $x + k$, then the answer will be greater than or equal to the correct one. By assumption, the correct answer is $\sum_{i=1}^{n} |a_i - x|$.

Since x is the median, we can rewrite it as

$$\sum_{i=1}^{\lfloor \frac{n}{2} \rfloor} x - a_i + \sum_{i=\lfloor \frac{n}{2} \rfloor + 1}^{n} a_i - x$$

Now let's write the answer for $x + k$.

$$\sum_{i=1}^{n} |a_i - x - k| = \sum_{i=1}^{y} x + k - a_i + \sum_{i=y+1}^{n} a_i - x - k$$

where y is the index of the last element that is less than or equal to $x + k$. If $k > 0$, then $y \geq \frac{n}{2}$ and subtracting 2 from the answers, we get

$$\sum_{i=1}^{\lfloor \frac{n}{2} \rfloor} x - a_i + \sum_{i=\lfloor \frac{n}{2} \rfloor + 1}^{n} a_i - x - \sum_{i=1}^{y} x + k - a_i - \sum_{i=y+1}^{n} a_i - x - k =$$

$$\sum_{i=1}^{\lfloor \frac{n}{2} \rfloor} x - a_i - x - k + a_i + \sum_{i=\lfloor \frac{n}{2} \rfloor + 1}^{y} a_i - x - x - k - a_i + \sum_{i=y+1}^{n} a_i - x - a_i + x + k =$$

$$\sum_{i=1}^{\lfloor \frac{n}{2} \rfloor} -k + \sum_{i=\lfloor \frac{n}{2} \rfloor + 1}^{y} -2 \cdot x - k + \sum_{i=y+1}^{n} k =$$

$$-k \cdot \lfloor \frac{n}{2} \rfloor - 2 \cdot x \cdot (y - \lfloor \frac{n}{2} \rfloor) - k \cdot (y - \lfloor \frac{n}{2} \rfloor) + k \cdot (n - y) = -2 \cdot x \cdot y + 2 \cdot x \cdot \lfloor \frac{n}{2} \rfloor - 2 \cdot k \cdot y + k \cdot n$$

We know that $y \geq \frac{n}{2}$, so $-2 \cdot x \cdot y + 2 \cdot x \cdot \lfloor \frac{n}{2} \rfloor \leq 0$ and $-2 \cdot k \cdot y + k \cdot n \leq 0$, so the answer in this case is greater than or equal to. For the case when $k < 0$, the solution is symmetric.